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Chaos and avoided level crossings

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The influence of chaos on tunneling in the quantum phase space of the harmonically driven pendulum is studied herein. We analyze the avoided level crossing between a quasienergy state associated with the chaotic part of the phase space and a member of the nearly degenerate doublet localized on the symmetric Kolmogorov-Arnold-Moser (KAM) islands. As a result of the interaction the quasienergy states "exchange" their structure. The initially chaotic state evolves into the regular one and vice versa. This interchange significantly affects dynamical tunneling of wave packets centered on the KAM islands.

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Despite significant recent progress relatively little is known about the effects of chaos on a purely quantum mechanical process such as tunneling. Herein we adopt a broader definition of tunneling, which comprises not only the conventional penetration of a classically insurmountable potential barrier but also the quantum motion between classically disconnected phase space nonlinear resonances [Kolmogorov-Arnold-Moser (KAM) islands]. The latter phenomenon was investigated by Davis and Heller as early as 1981 and is frequently referred to as dynamical tunneling [1].

The interplay between chaos and tunneling was the subject of the recent work by Lin and Ballentine [2,3] who investigated a monochromatically driven double-well potential. Lin and Ballentine show numerically that the tunneling between the KAM islands is 10^4 faster than that for the undriven case. Peres [4] pointed out that the observed tunneling is due to a dynamical symmetry of the Hamiltonian, which remains invariant under combined spatial reflection and time translation. Consequently, the eigenfunctions of the corresponding Floquet operator may be classified into even and odd states with respect to a generalized parity operator. In further investigations, Plata and Llorente [5] demonstrated that the tunneling rate is determined by the splitting of a pair of Floquet states localized on two stability regions. Utermann, Dittrich, and Hänggi [6] found very strong correlation between the level splitting and the overlap of the Husimi distribution of the doublet states with the chaotic layer. The behavior of the splitting of quasidegenerate doublets has also been the subject of a recent paper by Bohigas, Tomsovic, and Ullmo [7]. Using the autonomous system of two coupled quartic oscillators, they demonstrated that tunneling is strongly affected by classical integrability of the Hamiltonian system. Beyond the quasi-integrable regime, the splitting becomes extremely sensitive to variations of the external parameter. To quantify the observed phenomenon they considered a three-level model, which describes the interaction of the chaotic eigenstate with the quasidegenerate doublet. They concluded that "the major consequence of chaos is enhanced tunneling between islands by allowing transport across regions in phase space." The above scenario is valid only for sufficiently small coupling between the regular doublet and the chaotic state; a condition that may be satisfied far enough from the center of the avoided crossing. In the neighborhood close to the center of the crossing, quantum dynamics is very different. The purpose of this paper is to show that this difference is associated with the interchange of the structure of the interacting states—a generic quantum property of a two-level system [8].

As our model system we choose the driven pendulum [9], $H=p^2/(2\mu)+\mu(1+\cos q)-\mu\gamma q\cos(\Omega t)$, where γ is the peak amplitude of the driving force and Ω is its frequency. In all classical and quantum calculations discussed herein $\Omega=2$ and $\mu=5$. Performing the trivial scaling of the angular momentum by the parameter μ, $p'=p/\mu$ the equations of motion generated by the above Hamiltonian may be expressed in a form independent of μ. In the quantum domain, this scaling parameter plays a much more significant role since it may be interpreted as the inverse of the "effective" Planck's constant. The angle q varies between 0 and 2π contrary to the conventional range $[-\pi,\pi]$. This modification was introduced to facilitate quantum calculations. The Hamiltonian remains invariant under the following set of symmetry operations: $q\rightarrow -q$, $t\rightarrow t+\pi/\Omega$. This dynamical symmetry is identical to that of the driven double-well potential and we expect that the generic properties of the driven pendulum holds true for a large class of bounded Hamiltonian systems exhibiting this kind of symmetry.

Employing the $\hat{p}\cdot\hat{A}$ gauge, the solution of the time-dependent Schrödinger equation corresponding to H may be formally written as

$$|\psi(t)\rangle = \exp_T\left[-i\int_0^t\left\{\frac{1}{2\mu}[\hat{p}-\hat{A}(t')]^2 + \mu(1+\cos\hat{q})\right\}dt'\right]|\psi(0)\rangle\ , \quad (1)$$

where $\hat{A}(t) = -\mu\gamma\sin(\Omega t)/\Omega$ and Planck's constant was set to unity. In the most straightforward approach, the wave function is expanded in the eigenbasis of the angular momentum operator $\hat{p}$: $|\phi_n\rangle = \exp(inq)/\sqrt{2\pi}$ with coefficients $c_n(t)$. Furthermore, the time dependence of the vector potential $\hat{A}(t)$ is approximated by [10]

$$\hat{A}(t) = -\Delta t\mu\gamma\frac{\sin(\Omega t)}{\Omega}\sum_k\delta(t-k\Delta t)\ , \quad (2)$$

with $\Delta t = 2\pi/(\Omega L)$, where L is the number of integration steps taken per period T of the driving force. Then the integration in (1) may be carried out analytically and the vector of the expansion coefficients $\bar{c}(t_k+\Delta t)$ at time $t_k+\Delta t$ may be obtained by the successive multiplication of vector $\bar{c}(t_k)$ at time t_k by unitary matrices: $\bar{c}(t_k+\Delta t) = UQVQ^{-1}\bar{c}(t_k)$. U and V are both diagonal matrices with $U_{n,n} = \exp[-i\Delta t\{n^2/(2\mu)+n\gamma\sin(\Omega t_k)/\Omega\}]$ and $V_{n,n} = \exp[-i\Delta t v_n]$, v_n are the eigenvalues of the matrix representation N of the operator $\mu\cos\hat{q}$ in the eigenbasis of the operator $\hat{p}$, and Q is a unitary matrix that transforms N into diagonal form $(N = QVQ^{-1})$.

In Fig. 1 we present a small portion of the Floquet spectrum [11] of the driven pendulum. Note that the Floquet states may be classified into states of even and odd parity with respect to the previously mentioned generalized parity transformation. One can see from Fig. 1 that a typical double cone structure of the avoided level crossing originates as a result of interaction between quasienergy state A and a member of nearly degenerate doublet B, both having the same symmetry. The other member of the doublet, labeled in Fig. 1 as C, has a dynamical symmetry opposite those of states A and B. If one approximates the observed crossing with the help of a three-level model, then the selection rules reduce the problem to the interaction between states A and B.

To elucidate the nature of the avoided level crossing displayed in Fig. 1, let us examine the connection between the classical phase space portrait of the driven pendulum and the structure of the Floquet states A, B, and C. Figure 2 shows the Poincaré surface of section calculated for $\gamma = 4.92$ (cf. Fig. 1). Note that the KAM tori corresponding to oscillations around the elliptic fixed point (0,0) of the unperturbed Hamiltonian have been destroyed and now are part of a chaotic sea. Even for such a large value of perturbation strength, there are still high-order nonlinear resonances immersed in the chaotic sea. The regular part of the phase space is delineated by the noticeably deformed KAM tori found at $|p| \approx 4.2$. With the growing absolute value of the angular momentum, tori become less distorted and closely resemble those of unperturbed rotations. In Fig. 3 we present the contour plots of the Husimi distributions [12,13] of the quasienergy states A, B, and C obtained for the same value of the amplitude γ as that used in Fig. 2. Six contour lines uniformly spaced between zero and the maximum value were used. Note that each contour plot was superimposed with the selected tori taken from Fig. 2 (heavy dots). For clarity, the symmetric counterparts of these tori have not been shown. The "chaotic" character of the Floquet state A is clearly seen in Fig. 3(a). Its Husimi distribution spreads all over the chaotic sea and only slightly overlaps the regular part of the phase space. The interpretation of this overlap shall be given subsequently. The nature of the other states B and C displayed in Figs. 3(b) and 3(c) is utterly different. We refer to these states as intermediate since they are localized on the border of the regular and stochastic component of the phase space. This type of quasienergy state is common for Hamiltonian systems with mixed phase spaces. Although the distributions from Figs. 3(b) and 3(c) are similar to each other, as one would expect from the states that form the nearly degenerate doublet, state B clearly penetrates

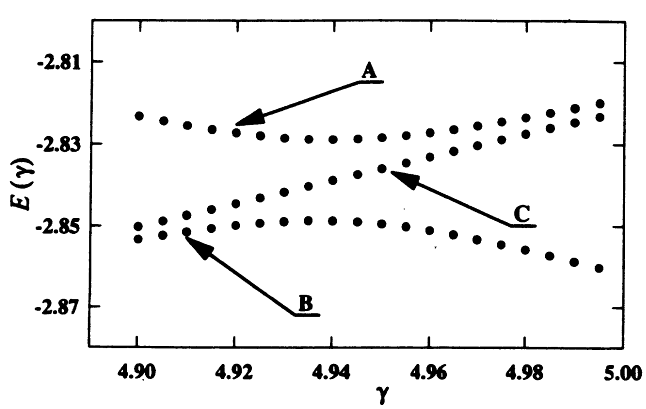

FIG. 1. The small portion of the Floquet spectrum of the driven pendulum shows the avoided crossing between the state A and a member of doublet B.

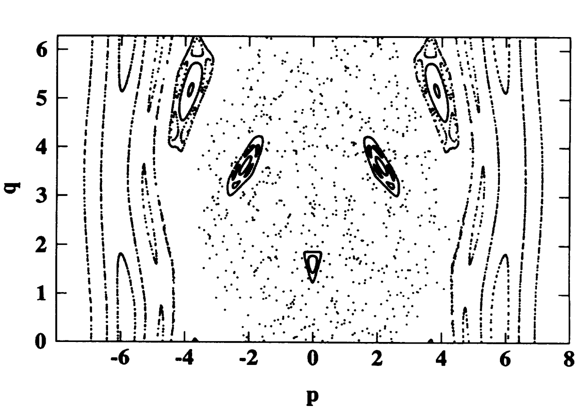

FIG. 2. Poincaré surface of section for the driven pendulum with $\Omega = 2$, $\mu = 5$, and $\gamma = 4.92$. Momentum was scaled by the parameter μ.

the chaotic part of the phase space, whereas C does not.

We have already mentioned that the repulsion between states A and B may be well approximated by a simple two-level model. The contamination of the Husimi distribution of state B in Fig. 3(b) by a chaotic component is a precursor of the complete exchange of the structure between initially regular (localized) state B and originally chaotic (delocalized) state A. This purely quantum mechanical process, characteristic of a two-level system [8], is elucidated in Fig. 4. In this figure, the expansion of all three states in the basis of eigenfunction $|\phi_n\rangle$ of the angular momentum operator are plotted for five values of the amplitude γ chosen from the interval [4.92,4.96]. The inspection of Fig. 1 shows that this interval comprises the main part of the avoided level crossing. In all the graphs in Fig. 4, the quantum number n varies between -30 and 30, the range of occupation probability is [0,0.2]. From the first panel in Fig. 4, which corresponds to the amplitude $\gamma=4.92$, it is apparent that the repulsion has already influenced the structure of both states A and B. The latter one is still similar to its doublet counterpart C (we have already pointed out this fact during the analysis of Fig. 3), but now populates the basis states that span the initially chaotic state A. On the other hand, the overlap of state A, initially spread in the classically stochastic region, with the regular part of the phase space is now easily understood as a simple quantum property of a system of two interacting levels [8].

The tunneling in phase space produced by the nearly degenerate doublet is destroyed by this same interaction mechanism. We can see in Fig. 4 that with the growing perturbation the mixing of states A and B becomes more strongly pronounced, so that at $\gamma=4.94$ they look very much alike. While bearing some resemblance to state C, which as expected from the symmetry analysis is not affected by the crossing and slowly varies in the range of perturbation shown in Fig. 1, they are delocalized in the

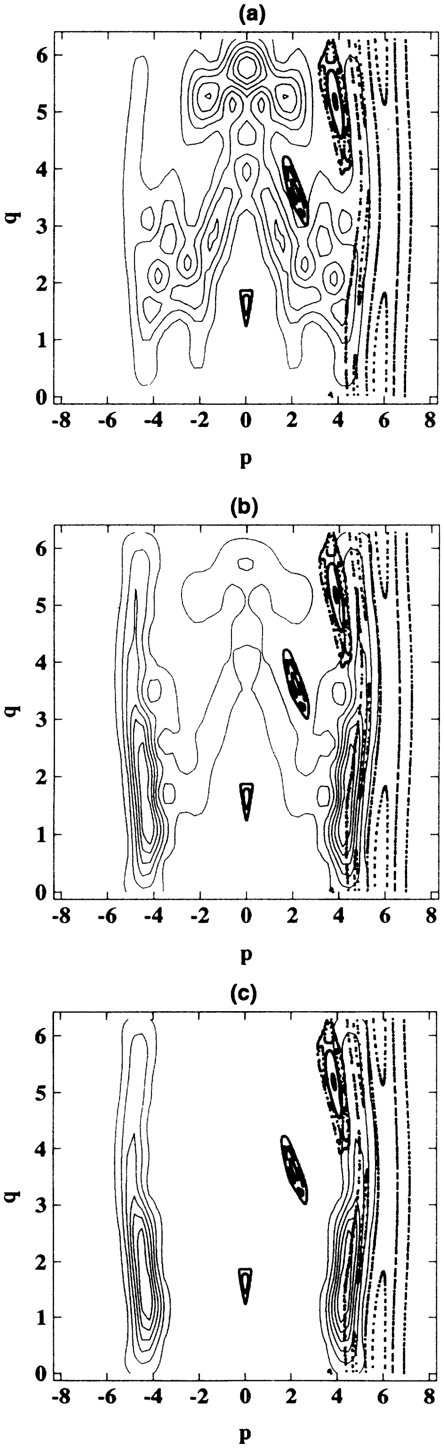

FIG. 3. Husimi representation of the Floquet states shown in Fig. 1. Momentum was scaled by the parameter μ. The model parameters are the same as in Fig. 2. The contour plots were superimposed with selected tori from Fig. 2. (a) corresponds to quasienergy state A, (b) to B, and (c) to C.

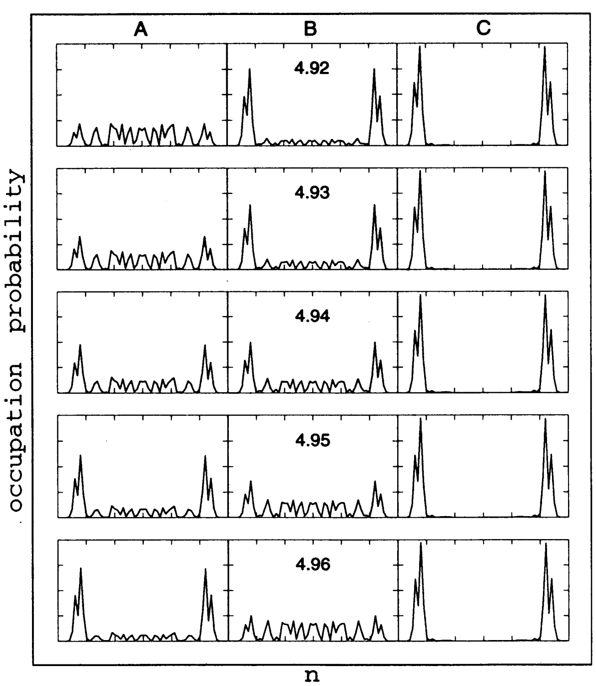

FIG. 4. Basis expansion of Floquet states A, B, and C plotted for five values of the amplitude γ. In all graphs, the quantum number n varies between -30 and 30. The range of occupation probability is [0,0.2].

classically chaotic part of the phase space. At this point the linear combinations of states B and C no longer yield a wave packet localized in the symmetric parts of the phase space, the property which is the hallmark of phase space tunneling. We emphasize that herein we have focused our attention on the tunneling produced by the members of the nearly degenerate doublet. In a forthcoming paper [14], we shall show how the avoided level crossing described here gives rise to perfectly periodic dynamical tunneling involving three quasienergy states (the regular doublet and the chaotic state). We have found this type of tunneling to be particularly robust against any symmetry breaking perturbation.

If the amplitude of the driving force is further increased, the "rotation" of quasienergy states A and B proceeds and, at $\gamma=4.96$, they have completely exchanged their structures (cf. the first and the last panel in Fig. 4). The initially chaotic state A becomes fairly regular and the initially regular state B becomes chaotic. From Fig. 1 we can infer that with the growing perturbation the splitting between states A and C steadily decreases until the nearly degenerate doublet is made up of states A and C rather than B and C.

Now let us assess the role of classical chaos in the phenomenon discussed above. The initially chaotic state A originates in the part of the phase space which for small perturbation corresponds to oscillations of the nonlinear pendulum. With the growing perturbation, the original structure of quasienergy state A (its excellent semiclassical approximation may be obtained with the help of Einstein-Brillouin-Kramer quantization rules extended for the periodically perturbed systems by Breuer and Holthaus [15]) is destroyed and the state gradually delocalizes. The avoided level crossing does not take place until the spreading reaches the basis states, which span the nearly degenerate doublet. The KAM tori have been shown to persist in the quantum phase space as dynamical barriers, which inhibit wave functions from exploring classically forbidden regions of phase space [16,17]. For sufficiently small values of Planck's constant, the properties of the classical phase space are clearly reflected in the structure of the Floquet states. The kind of avoided level crossing displayed in Fig. 1 may be observed for sufficiently strong interaction between the chaotic state (or intermediate state, the case discussed in a forthcoming paper [14]) and the member of quasidegenerate doublet. This kind of interaction occurs only if the phase space representation of the doublet states are not entirely enclosed by the KAM tori and consequently are affected by the presence of the chaotic sea. We have not been able to observe any detectable change in a level splitting when a third regular state crosses the path of a doublet. The latter behavior was found by Bohigas, Tomsovic, and Ullmo in the autonomous system of coupled quartic oscillators [7]. Thus, we can see that chaos influences all levels involved in the avoided level crossing. We think that the term *chaos induced avoided level crossings* or *chaotic avoided level crossings* amply describes the nature of the discussed phenomenon.

Note added. After completion of this paper we became aware that a related phenomenon was found in the standard quantum map by G. Casati, B. Chirikov, G. Fusina, and F. Izrailev (unpublished).

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